

NATURAL MAPS DEPENDING ON REDUCTIONS OF FRAME BUNDLES

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ABSTRACT. We clarify how the natural transformations of fiber product preserving bundle functors on \mathcal{FM}_m can be constructed by using reductions of the r -th order frame bundle of the base, \mathcal{FM}_m being the category of fibered manifolds with m -dimensional bases and fiber preserving maps with local diffeomorphisms as base maps. The iteration of two general r -jet functors is discussed in detail.

Consider a fibered manifold $p: Y \rightarrow M$ and its iterated jet prolongation $J^s J^r Y = J^s(J^r Y \rightarrow M)$. M. Modugno, [12], constructed an involutive map $\text{ex}_\Lambda: J^1 J^1 Y \rightarrow J^1 J^1 Y$ depending on a classical connection Λ on M . In [9], Modugno and the author proved that the only natural transformation $J^1 J^1 Y \rightarrow J^1 J^1 Y$ is the identity and the only two natural transformations $J^1 J^1 Y \rightarrow J^1 J^1 Y$ depending on a torsion-free Λ are $\text{id}_{J^1 J^1 Y}$ and ex_Λ . Using the Weil algebra technique, M. Doupovec and the author deduced that the only natural transformation $J^r J^s Y \rightarrow J^r J^s Y$ is the identity, [1]. In [2], Doupovec and W. M. Mikulski constructed a map $J^r J^s Y \rightarrow J^s J^r Y$ depending naturally on Λ . Mikulski, [11], discussed the natural transformations of two fiber product preserving bundle (in short: f.p.p.b.) functors on \mathcal{FM}_m depending on Λ .

We present a certain generalization of the last result by Mikulski. First of all, we point out that our main idea appears already in the case of classical natural bundles over m -manifolds. In Section 1, we consider two such bundles $F_1 M = P^r M[S_1]$, $F_2 M = P^r M[S_2]$ and a map $\varphi: S_1 \rightarrow S_2$ that is K -equivariant with respect to a subgroup $K \subset G_m^r$ only. Then every reduction Q of $P^r M$ to K determines a map $\varphi_Q: F_1 M \rightarrow F_2 M$ that is natural in Q . In the most important cases, K

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is the classical injection of G_m^1 into G_m^r . Here we find an interesting application of our result from [3], see also [5], that the reductions of $P^r M$ to G_m^1 are in bijection with the torsion-free connections on $P^{r-1}M$. In particular, every classical torsion-free connection Λ on M defines such reduction by means of the exponential map. As a simple illustration, we determine all vector bundle morphisms $J^1TM \rightarrow TM \otimes T^*M$ depending naturally on Λ .

Even Section 2 is of auxiliary character. We consider a bundle functor E on the product category $\mathcal{M}f_m \times \mathcal{M}f$ preserving products in the second factor. According to [8], these functors are determined by a Weil algebra and a group homomorphism $H: G_m^r \rightarrow \text{Aut } A$, where $\text{Aut } A$ is the group of all algebra automorphisms of A . Proposition 1 states that for two such functors (A_i, H_i) , $i = 1, 2$, and $K \subset G_m^r$, an K -equivariant algebra homomorphism $\mu: A_1 \rightarrow A_2$ and a K -reduction Q of $P^r M$ determine a map $\varphi_{Q,N}: (A_1, H_1)(M, N) \rightarrow (A_2, H_2)(M, N)$ that is natural in Q .

In Section 3 we consider the general case of a f.p.p.b. functor F of $\mathcal{F}\mathcal{M}_m$. Here we take into account the injection of categories $i: \mathcal{M}f_m \times \mathcal{M}f \rightarrow \mathcal{F}\mathcal{M}_m$ transforming (M, N) into the product fibered manifold $M \times N \rightarrow M$. According to [8], F is determined by $\bar{F} = F \circ i = (A, H)$ and an equivariant algebra homomorphism $t: \mathbb{D}_m^r \rightarrow A$, $\mathbb{D}_m^r = J_0^r(\mathbb{R}^m, \mathbb{R})$. Proposition 2 reads that for a fibered manifold $Y \rightarrow M$, $\dim M = m$, and two such functors $F_i = (A_i, H_i, t_i)$, $i = 1, 2$, every K -equivariant algebra homomorphism $\mu: A_1 \rightarrow A_2$ satisfying $t_2 = \mu \circ t_1$ and every K -reduction $Q \subset P^r M$ determine a map $\tilde{\mu}_{Q,Y}: F_1 Y \rightarrow F_2 Y$ that is natural in Q . In the last section, we deduce by Proposition 2 that there exists an exchange map of the iteration of two general non-holonomic jet functors depending naturally on a classical torsion-free connection on the base.

All manifolds and maps are assumed to be infinitely differentiable. Unless otherwise specified, we use the terminology and notation from the book [7].

1. THE CASE OF $\mathcal{M}f_m$

The classical natural bundles over m -manifolds, i.e. the bundle functors on the category $\mathcal{M}f_m$ of m -dimensional manifolds and local diffeomorphisms, are in bijection with the left actions $l: G_m^r \times S \rightarrow S$, where G_m^r means the r -th jet group in dimension m , [7]. For every m -manifold M , FM is the associated bundle $P^r M[S, l]$ to the r -th order frame bundle $P^r M$ and, for a local diffeomorphism $f: M \rightarrow M'$, we have $Ff = P^r f[\text{id}_S]$, i.e. $Ff(\{u, z\}) = \{P^r f(u), z\}$, $u \in P^r M$, $z \in S$. If $F_i M = P^r M[S_i, l_i]$, $i = 1, 2$, then natural transformations

$F_1 \rightarrow F_2$ are in bijection with G_m^r -maps $\varphi: S_1 \rightarrow S_2$. The induced map $\varphi_M: F_1M \rightarrow F_2M$ is of the form

$$\varphi_M(\{u, z\}) = \{u, \varphi(z)\}, \quad u \in P^rM, \quad z \in S_1.$$

We are interested in the case we have a subgroup $K \subset G_m^r$ and $\varphi: S_1 \rightarrow S_2$ is K -equivariant only. Then we can use a reduction $Q \subset P^rM$ to K . Both F_iM can be interpreted as associated bundles to Q , i.e. $F_iM = Q[S_i]$, $i = 1, 2$, and we define

$$\varphi_Q: Q[S_1] \rightarrow Q[S_2], \quad \varphi_Q(\{u, z\}) = \{u, \varphi(z)\}, \quad u \in Q, \quad z \in S_1.$$

This is a correct definition, for

$$(1) \quad \begin{aligned} \varphi_Q(\{u \circ k, l_1(k^{-1})(z)\}) &= \{u \circ k, \varphi(l_1(k^{-1})(z))\} \\ &= \{u \circ k, l_2(k^{-1})(\varphi(z))\} = \varphi_Q(\{u, z\}), \quad k \in K, \end{aligned}$$

by K -equivariance of φ .

If we consider a K -reduction Q' of P^rM' that is f -related with Q , i.e. $P^rf(u) \in Q'$ for all $u \in Q$, then the following diagram commutes

$$(2) \quad \begin{array}{ccc} F_1M & \xrightarrow{\varphi_Q} & F_2M \\ F_1f \downarrow & & \downarrow F_2f \\ F_1M' & \xrightarrow{\varphi_{Q'}} & F_2M' \end{array}$$

Indeed, $F_2f(\{u, \varphi(z)\}) = \{P^rf(u), \varphi(z)\}$. So we say that the maps φ_Q are natural with respect to the choice of K -reductions.

The most interesting case is $K = \iota_r(G_m^1)$, where $\iota_r: G_m^1 \rightarrow G_m^r$ is the standard injection $\iota_r(a) = j_0^r \tilde{a}$, $\tilde{a}: \mathbb{R}^m \rightarrow \mathbb{R}^m$ being the linear map determined by $a \in G_m^1$. Every classical torsion-free connection Λ on M defines a reduction $\exp_\Lambda^{r-1}: P^1M \rightarrow P^rM$ to $\iota_r(G_m^1)$ as follows, [5]. The exponential map $\exp_{\Lambda, x}$ of Λ at x is a local map of T_xM into M , $u \in P_x^1M$ can be interpreted as a linear map $\tilde{u}: \mathbb{R}^m \rightarrow T_xM$ and we define

$$(3) \quad \exp_\Lambda^{r-1}(u) = j_0^r(\exp_{\Lambda, x} \circ \tilde{u}) \in P_x^rM.$$

If $Q = \exp_\Lambda^{r-1}(P^1M)$, we say that $\varphi_Q =: \varphi_\Lambda$ is determined by Λ .

We recall there is a canonical $(\mathbb{R}^m \times \mathfrak{g}_m^{r-1})$ -valued one-form θ_r on P^rM and the torsion of a connection Γ on P^rM is the covariant exterior differential $D_\Gamma \theta_r$. In [3], see also [5], we deduced the following assertion.

Lemma 1. *There is a canonical bijection between torsion-free connections on $P^{r-1}M$ and the reductions of P^rM to $\iota_r(G_m^1)$.*

Example. To illustrate this procedure, we consider a very simple case $J^1TM \rightarrow TM \otimes T^*M$. Then S_1 is G_m^2 -space $\mathbb{R}^m \times \mathbb{R}^m \otimes \mathbb{R}^{m*}$ and S_2 is G_m^1 -space $\mathbb{R}^m \otimes \mathbb{R}^{m*}$ interpreted as a G_m^2 -space by means of the jet projection $G_m^2 \rightarrow G_m^1$. We evaluate directly that all G_m^2 -maps $S_1 \rightarrow S_2$ are the constant maps of S_1 into $k \text{id}_{\mathbb{R}^m}$, $k \in \mathbb{R}$. In the case of $\iota_2(G_m^1) \subset G_m^2$, it is reasonable to restrict ourselves to the linear G_m^1 -maps $S_2 \rightarrow S_1$. We find directly that all of them are of the form

$$(4) \quad x_j^i = c_1 y_j^i + c_2 \delta_j^i y_k^k, \quad c_1, c_2 \in \mathbb{R},$$

$(y^i, y_j^i) \in S_1, x_j^i \in S_2$. In some local coordinates x^i on M , the geodesics of Λ are determined by

$$(5) \quad \frac{d^2 x^i}{dt^2} + \Lambda_{jk}^i(x) \frac{dx^j}{dt} \frac{dx^k}{dt} = 0.$$

Every frame from $\exp_\Lambda^1(P_x^1 M)$ is characterized by $\Lambda_{jk}^i(x) = 0$. Hence (4) implies that all vector bundle morphisms $J^1TM \rightarrow TM \otimes T^*M$ determined by Λ form the following 2-parameter family

$$(6) \quad (j_x^1 s) \mapsto c_1 (\nabla_\Lambda s)(x) + c_2 \text{Ctr}((\nabla_\Lambda s)(x)) \text{id}_{T_x M}, \quad c_1, c_2 \in \mathbb{R},$$

where $\nabla_\Lambda s$ is the covariant differential of a section s of TM with respect to Λ and $\text{Ctr}((\nabla_\Lambda s)(x))$ means the contraction of this $(1, 1)$ -tensor.

2. THE CASE OF $\mathcal{M}f_m \times \mathcal{M}f$

According to [8], if we intend to study a f.p.p.b. functor F on \mathcal{FM}_m , we first have to discuss a bundle functor E on the product category $\mathcal{M}f_m \times \mathcal{M}f$ that preserves products in the second factor, i.e.

$$E(M, N_1 \times N_2) = E(M, N_1) \times_M E(M, N_2).$$

These functors are identified with pairs $E = (A, H)$ of a Weil algebra A and a group homomorphism $H: G_m^r \rightarrow \text{Aut } A$. In general, every algebra homomorphism $\mu: A_1 \rightarrow A_2$ of two Weil algebras determines a natural transformation $\mu_M: T^{A_1}M \rightarrow T^{A_2}M$ of the corresponding Weil functors, [4], [7]. This defines an action $H_N: g \mapsto H(g)_N$ of G_m^r on $T^A N$ and $E(M, N)$ is the corresponding associated bundle

$$(A, H)(M, N) = P^r M[T^A N, H_N].$$

If $f_1: M \rightarrow M'$ is a local diffeomorphism and $f_2: N \rightarrow N'$ is a smooth map, we have

$$(7) \quad (A, H)(f_1, f_2)(\{u, Z\}) = \{P^r f_1(u), T^A f_2(Z)\}, \quad u \in P^r M, Z \in T^A N.$$

For two such functors $E_i = (A_i, H_i)$, $i = 1, 2$, the natural transformations $E_1 \rightarrow E_2$ are in bijection with G_m^r -equivariant algebra homomorphisms $\mu: A_1 \rightarrow A_2$, i.e.

$$(8) \quad \mu(H_1(g)(a)) = H_2(g)(\mu(a)), \quad a \in A_1, g \in G_m^r,$$

[8]. The induced map $\mu_{M,N}: (A_1, H_1)(M, N) \rightarrow (A_2, H_2)(M, N)$ is of the form

$$(9) \quad \mu_{M,N}(\{u, Z\}) = \{u, \mu_N(Z)\}, \quad u \in P^r M, Z \in T^{A_1} N.$$

Consider the case μ is K -equivariant, i.e. (8) holds for $g \in K \subset G_m^r$ only. If we take a K -reduction $Q \subset P^r M$, we may write $E_i(M, N) = Q[T^{A_i} N]$, $i = 1, 2$, and we can define $\mu_{Q,N}: E_1(M, N) \rightarrow E_2(M, N)$ by

$$(10) \quad \mu_{Q,N}(\{u, Z\}) = \{u, \mu_N(Z)\}, \quad u \in Q, z \in T^{A_1} N.$$

Since μ is K -equivariant, this definition is correct. Further, if Q' is an f_1 -related K -reduction of $P^r M'$, then one verifies analogously to Section 1 that the following diagram commutes

$$(11) \quad \begin{array}{ccc} (A_1, H_1)(M, N) & \xrightarrow{(A_1, H_1)(f_1, f_2)} & (A_1, H_1)(M', N') \\ \downarrow \mu_{Q,N} & & \downarrow \mu_{Q',N'} \\ (A_2, H_2)(M, N) & \xrightarrow{(A_2, H_2)(F_1, f_2)} & (A_2, H_2)(M', N') \end{array}$$

Thus, we have proved

Proposition 1. *For two functors $E_i = (A_i, H_i)$, $i = 1, 2$, with the same r and m and a subgroup $K \subset G_m^r$, every K -equivariant algebra homomorphism $\mu: A_1 \rightarrow A_2$ and every K -reduction $Q \subset P^r M$ determine a map $\mu_{Q,N}: E_1(M, N) \rightarrow E_2(M, N)$ that is natural in the sense of (11).*

3. THE CASE OF \mathcal{FM}_m

We have an injection of categories $i: \mathcal{M}f_m \times \mathcal{M}f \rightarrow \mathcal{FM}_m$ transforming (M, N) into the product fibered manifold $M \times N \rightarrow M$ and (f_1, f_2) into the product \mathcal{FM}_m -morphism $f_1 \times f_2$ with base map f_1 . If F is a f.p.p.b. functor on \mathcal{FM}_m , then $\bar{F} := F \circ i$ is a bundle functor on $\mathcal{M}f_m \times \mathcal{M}f$ that preserves products in the second factor, so that $\bar{F} = (A, H)$. According to [8], F is identified with a triple $F = (A, H, t)$, where $t: \mathbb{D}_m^r \rightarrow A$ is an equivariant algebra homomorphism. Hence $t_M: T_m^r M \rightarrow T^A M$. For a fibered manifold $p: Y \rightarrow M$, we have $FY \subset \bar{F}(M, Y) = P^r M[T^A Y]$ and

$$(12) \quad \{u, Z\} \in FY \text{ means } t_M(u) = T^A p(Z) \in T^A M, \quad u \in P^r M, \quad Z \in T^A Y,$$

where $T^A p: T^A Y \rightarrow T^A M$ and we use $P^r M \subset T_m^r M$. Let $p': Y' \rightarrow M'$ be another fibered manifold, $\dim M' = m$, and $f: Y \rightarrow Y'$ be an \mathcal{FM}_m -morphism with base map $\underline{f}: M \rightarrow M'$. Then $Ff: FY \rightarrow FY'$ is the restriction and corestriction of $\bar{F}(\underline{f}, f): \bar{F}(M, Y) \rightarrow \bar{F}(M', Y')$.

Consider $F_i = (A_i, H_i, t_i)$, $i = 1, 2$. According to [8], the natural transformations $F_1 \rightarrow F_2$ are in bijection with G_m^r -equivariant algebra homomorphisms $\mu: A_1 \rightarrow A_2$ satisfying $t_2 = \mu \circ t_1$. The corresponding map $\tilde{\mu}_Y: F_1 Y \rightarrow F_2 Y$ is of the form

$$(13) \quad \tilde{\mu}_Y(\{u, Z\}) = \{u, \mu_Y(Z)\}, \quad u \in P^r M, \quad Z \in T^{A_1} Y,$$

where $\mu_Y: T^{A_1} Y \rightarrow T^{A_2} Y$ is the map determined by μ in the manifold case.

Assume again that μ is K -equivariant only, $K \subset G_m^r$, and we have a K -reduction $Q \subset P^r M$. Then $F_i Y \subset Q[T^{A_i} Y]$, $i = 1, 2$, and there is a restricted and corestricted map $\tilde{\mu}_{Q,Y}$

$$(14) \quad \begin{array}{ccc} F_1 Y & \xrightarrow{\tilde{\mu}_{Q,Y}} & F_2 Y \\ \downarrow & & \downarrow \\ Q[T^{A_1} Y] & \xrightarrow{\mu_{Q,Y}} & Q[T^{A_2} Y] \end{array}$$

Analogously to Section 2, one verifies directly that the following diagram commutes

$$(15) \quad \begin{array}{ccc} F_1 Y & \xrightarrow{\tilde{\mu}_{Q,Y}} & F_2 Y \\ F_1 f \downarrow & & \downarrow F_2 f \\ F_1 Y' & \xrightarrow{\tilde{\mu}_{Q',Y'}} & F_2 Y' \end{array}$$

Thus, we have proved

Proposition 2. *For two functors $F_i = (A_i, H_i, t_i)$, $i = 1, 2$, with the same r and m and a subgroup $K \subset G_m^r$, every K -equivariant algebra homomorphism $\mu: A_1 \rightarrow A_2$ satisfying $t_2 = \mu \circ t_1$ and every K -reduction $Q \subset P^r M$ determine a map $\tilde{\mu}_{Q,Y}: F_1 Y \rightarrow F_2 Y$ that is natural in the sense of (15).*

4. ITERATION OF GENERAL NONHOLONOMIC JET FUNCTORS

The r -th nonholonomic prolongation $\tilde{J}^r Y$ of Y is introduced by the iteration $\tilde{J}^r Y = J^1(\tilde{J}^{r-1} Y \rightarrow M)$, $\tilde{J}^1 Y = J^1 Y$. The bundle $\tilde{J}^r(M, N)$ of nonholonomic r -jets of M into N is defined as $\tilde{J}^r(M \times N \rightarrow M)$. We have $J^r Y \subset \tilde{J}^r Y$ and $J^r(M, N) \subset \tilde{J}^r(M, N)$. The composition $Z \circ X \in \tilde{J}_x^r(M, Q)_z$ of $X \in \tilde{J}_x^r(M, N)_y$ and $Z \in \tilde{J}_y^r(N, Q)_z$ coincides

with the classical one for holonomic r -jets, [4]. We write $\beta_Y^r: \tilde{J}^r Y \rightarrow Y$ for the target jet projection. If we consider $\tilde{J}^s \tilde{J}^r Y = \tilde{J}^s(\tilde{J}^r Y \rightarrow M)$, we have the target projection $\beta_{\tilde{J}^r Y}^s: \tilde{J}^s \tilde{J}^r Y \rightarrow \tilde{J}^r Y$ and the induced map $\tilde{J}^s \beta_Y^r: \tilde{J}^s \tilde{J}^r Y \rightarrow \tilde{J}^s Y$. A map $e: \tilde{J}^s \tilde{J}^r Y \rightarrow \tilde{J}^r \tilde{J}^s Y$ is called an exchange, if $\tilde{J}^r \beta_Y^s \circ e = \beta_{\tilde{J}^r Y}^s$ and $\beta_{\tilde{J}^s Y}^r \circ e = \tilde{J}^s \beta_Y^r$. We use Proposition 2 to prove the following assertion.

Proposition 3. *Every classical torsion-free connection Λ on M determines an exchange map $\text{ex}_\Lambda: \tilde{J}^s \tilde{J}^r Y \rightarrow \tilde{J}^r \tilde{J}^s Y$.*

Proof. According to the iteration theory, [1], the Weil algebra $\tilde{\mathbb{D}}_m^r$ of \tilde{J}^r is the tensor product $\tilde{\mathbb{D}}_m^r = \otimes_m^r \mathbb{D}_m^1$, $\mathbb{D}_m^1 = J_0^1(\mathbb{R}^m, \mathbb{R}) = \mathbb{R} \times \mathbb{R}^{m*}$. The corresponding action of $\iota_r(G_m^1)$, on \mathbb{D}_m^r is the tensor product of the identities on \mathbb{R} and the classical actions of G_m^1 on \mathbb{R}^{m*} . By the iteration theory, the Weil algebra of $\tilde{J}^s \tilde{J}^r$ is $\tilde{\mathbb{D}}_m^s \otimes \tilde{\mathbb{D}}_m^r$ and the corresponding action of $\iota_{r+s}(G_m^1)$ is of the same type. Hence the exchange map $e: \tilde{\mathbb{D}}_m^s \otimes \tilde{\mathbb{D}}_m^r \rightarrow \tilde{\mathbb{D}}_m^r \otimes \tilde{\mathbb{D}}_m^s$ is $\iota_{r+s}(G_m^1)$ -equivariant. Using the exponential map of Λ , we construct ex_Λ . \square

Remark. We underline that for $r > 1$ or $s > 1$ there is no hope for the uniqueness result from the case $r = s = 1$ mentioned in the introduction. By Lemma 1, instead of \exp_Λ^{r+s-1} we can use any natural operator transforming Λ into a torsion-free connection on $P^{r+s}M$. All these operators were characterized by Mikulski, [10], with an addendum concerning the torsion-free case in [5].

This approach works even if we replace \tilde{J}^r by a general r -jet category C introduced in [6]. This is a rule transforming every pair (M, N) of manifolds into a fibered submanifold $C(M, N) \subset \tilde{J}^r(M, N)$ such that

- (i) $J^r(M, N) \subset C(M, N)$ is a fibered submanifold,
- (ii) if $X \in C_x(M, N)_y$ and $Z \in C_y(N, Q)_z$, then $Z \circ X \in C_x(M, Q)_z$,
- (iii) if $X \in C_x(M, N)_y$ is regular, i.e. there exists $W \in \tilde{J}_y^r(N, M)_x$ such that $W \circ X = j_x^r \text{id}_M$, then there exists $Z \in C_y(M, N)_x$ with this property,
- (iv) $C(M, N \times Q) = C(M, N) \times_M C(M, Q)$.

For every fibered manifold $p: Y \rightarrow M$, we define its horizontal C -prolongation

$$C_h Y = \{X \in C(M, Y), (j_{\beta_X}^r p) \circ X = j_{\alpha_X}^r \text{id}_M\},$$

αX or βX being the source or target of X , and its vertical C -prolongation

$$C_v Y = \bigcup_{x \in M} C_x(M, Y_x) \subset C(M, Y).$$

Clearly, $\tilde{J}_h^r Y = \tilde{J}^r Y$. If we restrict $C_h Y$ or $C_v Y$ to fibered manifolds with m -dimensional bases, we obtain a f.p.p.b. functor $C_{h,m}$ or $C_{v,m}$ on \mathcal{FM}_m , provided the values of $C_{h,m}$ or $C_{v,m}$ on \mathcal{FM}_m -morphisms are defined by means of the jet composition.

If we consider another general s -jet category C' , then the proof of Proposition 3 works for every pair from $C_{h,m}$, $C_{v,m}$, $C'_{h,m}$, $C'_{v,m}$.

REFERENCES

- [1] M. Doupovec, I. Kolář, *Iteration of fiber product preserving bundle functors*, Monatsch. Math. **134** (2001), 39–50.
- [2] M. Doupovec, W. M. Mikulski, *Higher order jet involution*, Czechoslovak Math. J. **57** (2007), 933–945.
- [3] I. Kolář, *Torsion-free connections on higher order frame bundles*, New Development in Differential Geometry, Proceedings, Kluwer (1996), 233–241.
- [4] I. Kolář, *Weil Bundles as Generalized Jet Spaces*, in: Handbook of Global Analysis, Elsevier (2008), 625–665.
- [5] I. Kolář, *Connections on higher order frame bundles and their gauge analogies*, in: Variations, Geometry and Physics, Nova Science Publishers (2008), 199–223.
- [6] I. Kolář, *On special types of nonholonomic contact elements*, to appear.
- [7] I. Kolář, P. W. Michor, J. Slovák, *Natural Operations in Differential Geometry*, Springer Verlag, 1993.
- [8] I. Kolář, W. M. Mikulski, *On the fiber product preserving bundle functors*, Differential Geom. Appl. **11** (1999), 105–115.
- [9] I. Kolář, M. Modugno, *Natural maps on the iterated jet prolongation of a fibered manifold*, Annali di Mat. pura ed applicata CLVIII (1991), 151–165.
- [10] W. M. Mikulski, *Higher order linear connections from first order ones*, Arch. Math. (Brno), **43** (2007), 285–288.
- [11] W. M. Mikulski, *On symmetrization of jets*, to appear in Czechoslovak Math. J.
- [12] M. Modugno, *Jet involution and prolongation of connections*, Čas. pěst. mat. **14** (1989), 356–365.

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