ON THE TORSION ON GAUGE-LIKE PROLONGATIONS OF PRINCIPAL BUNDLES

Ivan Kolář

Abstract. Every fiber product preserving bundle functor F on the category $\mathcal{FM}_m$ defines the gauge-like prolongation $W^F P$ of a principal bundle P, that coincides with the r-th principal prolongation $W^r P$ in the special case $F = J^r$. For a large class of such functors we introduce the torsion of connections on $W^F P$ and we deduce some its properties analogous to the case of $W^r P$.

The r-th principal (or gauge-natural) prolongation $W^r P \to M$ of a principal bundle $P \to M$ is a fundamental structure for both the theory of geometric object fields, [2], [9], and the gauge theories of mathematical physics, [3]. In [5] we introduced the torsion of a connection Γ on $W^r P$ to be the covariant exterior differential of the canonical one-form θ_r of $W^r P$. On the other hand, the Lie algebroid $L(W^r P)$ coincides with the r-th jet prolongation $J^r(LP)$ of the Lie algebroid LP of P, [11]. In [8], we considered the algebroid form $\gamma\colon TM \to J^r LP$ of Γ, we introduced the torsion of γ by using the truncated bracket of $J^r LP$ and we deduced that both approaches to the torsion are naturally equivalent.

In the present paper we study a more general setting of this problem. In [1], the authors constructed a principal bundle $W^F P \to M$ for every fiber product preserving bundle functor F on the category $\mathcal{FM}_m$ of fibered manifolds with m-dimensional bases and fibered morphisms with local diffeomorphisms as base maps. In the case of the functor J^r of r-th jet prolongation, we have $W^{J^r} P = W^r P$, so that $W^F P$ will be called a gauge-like prolongation of P. We are going to clarify that geometrically remarkable results concerning torsion appear in the case of a subfunctor $E \subset J^1 \circ F$. In this situation, we can use our results on the generalized G-structures on $W^F P$, i.e. the reductions of $W^1(W^F P)$, [4].

2000 *Mathematics Subject Classification*: 53C05, 53C10, 58A20, 58A32.
Key words and phrases: gauge-like prolongation of principal bundle, Lie algebroid, generalized G-structure, connection, torsion.
The author was supported by the Ministry of Education of Czech Republic under the project MSM 0021622409.

In Section 1 we first summarize the basic properties of $W^F P$. Then we construct a canonical map relating the gauge-like prolongation of the iteration of two functors with the iteration of the gauge-like prolongations. In Proposition 1 we determine the Lie algebroid version of this map. In Section 2 we study the reductions Q of $W^1 P$ (called generalized G-structures in [4]) from our point of view. In Proposition 2 we deduce that both approaches to the torsion on Q are naturally equivalent. Special attention is paid to the additional properties of semiprolongable generalized G-structures. Proposition 3 describes a relation between the prolongability of generalized G-structures and the existence of torsion-free connections analogous to the case of classical G-structures. In Section 3 we specify some fiber product preserving bundle functors on $\mathcal{FM}_m$, the gauge-like prolongations of which are of the form studied in Section 2. We also point out that some further functors can be reduced to this case by using a suitable natural equivalence.

All manifolds and maps are assumed to be infinitely differentiable. Unless otherwise specified, we use the terminology and notations from the book [9]. In particular, we write $P^r M$ for the r-th order frame bundle of a manifold M and G^r_m for the r-th jet group in dimension m. Under a connection we always mean a principal connection.

1. Gauge-like prolongations of principal bundles. We denote by T^A the Weil functor determined by a Weil algebra A, [9], [7]. A fundamental result reads that the product preserving bundle functors on the category $\mathcal{M}f$ of all manifolds are in bijection with the Weil functors and the natural transformations $h_M \colon T^{A_1} M \to T^{A_2} M$ are in bijection with the algebra homomorphisms $h \colon A_1 \to A_2$, [7]. In the special case of $\mathbb{D}^r_k = J^r_0(\mathbb{R}^k, \mathbb{R})$, $T^{\mathbb{D}^r_k} = T^r_k$ is the classical functor of (k, r)-velocities.

In [10], the authors deduced that the fiber product preserving bundle functors on $\mathcal{FM}_m$ of base order r are in bijection with the triples (A, H, t) of a Weil algebra A, a group homomorphism $H \colon G^r_m \to \mathrm{Aut}\, A$ and an equivariant algebra homomorphism $t \colon \mathbb{D}^r_m \to A$, where $\mathrm{Aut}\, A$ is the group of all algebra automorphisms of A and we take into account $\mathrm{Aut}\, (\mathbb{D}^r_m) = G^r_m$. One constructs the functor $F = (A, H, t)$ as follows. For every manifold N, we have an induced action H_N of G^r_m on $T^A N$,

$$H_N(g, z) = H(g)_N(z)\,, \quad g \in G^r_m,\ z \in T^A N\,,$$

where $H(g)_N \colon T^A N \to T^A N$ is the map determined by $H(g) \colon A \to A$. The value of F on the product fibered manifold $M \times N \to M$, $\dim M =$

m, is the associated bundle

$$(1)\qquad F(M\times N)=P^rM[T^AN,H_N]\,.$$

Every local diffeomorphism $f\colon M\to\bar M$ and every map $\varphi\colon N\to\bar N$ determine the product $\mathcal{FM}_m$-morphism $f\times\varphi\colon M\times N\to\bar M\times\bar N$. By naturality, the map $T^A\varphi\colon T^AN\to T^A\bar N$ is G^r_m-equivariant. Further, $P^rf\colon P^rM\to P^r\bar M$ is a principal bundle morphism. Then we define $F(f\times\varphi)\colon F(M\times N)\to F(\bar M\times\bar N)$ to be the morphism of associated bundles

$$(2)\qquad F(f\times\varphi)=(P^rf,T^A\varphi)\colon P^rM[T^AN,H_N]\to P^r\bar M[T^A\bar N,H_{\bar N}]\,.$$

For an arbitrary fibered manifold $p\colon Y\to M$, FY is the subbundle of $P^rM[T^AY,H_Y]$ of all elements $\{u,Z\}$, $u\in P^rM\subset T^r_mM$, $Z\in T^AY$ satisfying

$$(3)\qquad t_M(u)=T^Ap(Z)\,,$$

where $t_M\colon T^r_mM\to T^AM$ is the map induced by $t\colon\mathbb{D}^r_m\to A$. Since t is equivariant, (3) is independent of the choice of the representatives u and Z of the equivalence class $\{u,Z\}\in FY$. For another fibered manifold $\bar p\colon\bar Y\to\bar M$ and an $\mathcal{FM}_m$-morphism $f\colon Y\to\bar Y$ over $\underline{f}\colon M\to\bar M$, $(P^r\underline{f},T^Af)$ maps FY into $F\bar Y$. Then one defines Ff to be its restriction and corestriction. In the case of J^r, we have $A=\mathbb{D}^r_m$, $H=\mathrm{id}_{G^r_m}$, $t=\mathrm{id}_{\mathbb{D}^r_m}$ and (3) expresses, in fact, the classical relation

$$(4)\qquad J^rY=\big\{X\in J^r(M,Y);\ p_*(X)=j^r_x\,\mathrm{id}_M\big\}\,,$$

where x is the source of $X=j^r_xf$ and $p_*(X)=j^r_x(p\circ f)\in J^r(M,M)$.

Consider a principal bundle $P(M,G)$. First we recall $W^rP==P^rM\times_MJ^rP$, [9]. This is a principal bundle over M, whose structure group is the group semidirect product $W^r_mG=G^r_m\rtimes T^r_mG$ with the composition

$$(5)\qquad (g_1,C_1)(g_2,C_2)=\big(g_1\circ g_2,(C_1\circ g_2)\bullet C_2\big)\,,$$

where $\bullet$ denotes the induced group composition in T^r_mG. In [1], the authors defined

$$(6)\qquad W^FP=P^rM\times_MFP\,.$$

Analogously to (5), one constructs the group semidirect product

$$W^A_HG=G^r_m\rtimes_HT^AG$$

with the composition

$$(7)\qquad (g_1,C_1)(g_2,C_2)=\big(g_1\circ g_2,H(g_2^{-1})_G(C_1)\bullet C_2\big)\,,$$

where $\bullet$ denotes the induced group composition in T^AG. In the case of W^r_mG, $H(g_2^{-1})_G(C_1)=C_1\circ g_2$, so that (7) generalizes (5). Then

$W^F P$ is a principal bundle over M with structure group $W^A_H G$ with respect to the following action, [1]. The right action of G on P can be interpreted as an $\mathcal{FM}_m$-morphism

$$\varrho\colon P \times_M (M \times G) \to P\,.$$

Applying F, we obtain

$$F\varrho\colon FP \times_M P^r M[T^A G, H_G] \to FP\,.$$

For $(g, X) \in G^r_m \times T^A G$ and $(u, Y) \in P^r M \times_M FP$, one defines

$$(u, Y)(g, X) = \big(u \circ g, F\varrho(Y, \{u \circ g, X\})\big)\,. \tag{8}$$

In the case of $W^r P$, (8) coincides with the right action of $W^r_m G$ on $W^r P$ described in [9].

We remark that a basic geometric property of $W^F P$ is that for every bundle $D \to M$ associated to P, $FD \to M$ is a bundle associated to $W^F P$, [1].

Further, F determines a natural transformation $\tilde t_Y\colon J^r Y \to FY$. Every element $X \in J^r Y$ is of the form $j^r_x s$. We interpret the local section s of Y as a local $\mathcal{FM}_m$-morphism $\tilde s$ of the trivial fibered manifold $\mathrm{id}_M\colon M \to M$ into Y and we set

$$\tilde t_Y(X) = (F\tilde s)(x) \in FY\,. \tag{9}$$

In the product case, $J^r(M \times N) = P^r M[T^r_m N]$, $F(M \times N) = P^r M[T^A N]$ and $\tilde t_{M\times N}$ is of the form $\tilde t_{M\times N} = (\mathrm{id}_{P^r M}, t_N)$ with $t_N\colon T^r_m N \to T^A N$.

In particular, we have $\tilde t_{TM}\colon J^r TM \to FTM$. Write $q\colon LP \to TM$ for the anchor map, so that $Fq\colon FLP \to FTM$. In [6], we deduced that the Lie algebroid of $W^F P$ is

$$L(W^F P) = J^r TM \times_{FTM} FLP\,. \tag{10}$$

In the special case $F = J^r$, we reobtain

$$L(W^r P) = J^r TM \times_{J^r TM} J^r LP = J^r LP\,.$$

Consider two such functors F_1 and F_2 of base orders r and s. Then the base order of $F_2 \circ F_1$ is $r + s$. Using $(\tilde t_2)_{P^r M}\colon J^s P^r M \to F_2 P^r M$, we construct a map

$$W^{F_2\circ F_1} P \to W^{F_2}(W^{F_1} P) \tag{11}$$

as follows. The classical inclusion $P^{r+s} M \hookrightarrow W^s(P^r M) = P^s M \times_M J^s P^r M$ is described in [9]. So we have

$$\begin{aligned} W^{F_2\circ F_1} P = P^{r+s} M \times_M F_1 F_1 P &\hookrightarrow P^s M \times_M J^s P^r M \times_M F_2 F_1 P \\ &\to P^s M \times_M F_2 P^r M \times_M F_2 F_1 P \\ &\to P^s M \times_M F_2(W^{F_1} P) = W^{F_2}(W^{F_1} P)\,. \end{aligned} \tag{12}$$

According to [1], this is a principal bundle morphism. If t_2 is injective, (11) is an injection.

To construct the corresponding Lie algebroid homomorphism, we start with a general formula. Consider two principal bundles $P_1 \to M_1$ and $P_2 \to M_2$ over m-manifolds and a $\mathcal{PB}$-morphism $f\colon P_1 \to P_2$ over a local diffeomorphism $\underline{f}\colon M_1 \to M_2$. Write $Lf\colon LP_1 \to LP_2$ for the induced algebroid morphism. Using trivializations, one finds easily that the algebroid morphism $LW^F f\colon LW^F P_1 \to LW^F P_2$ is

$$(13)\quad J^r T\underline{f} \times_{FT\underline{f}} FLf\colon J^r TM_1 \times_{FTM_1} FLP_1 \to J^r TM_2 \times_{FTM_2} FLP_2\,.$$

Further, the algebroid form of the injection $P^{r+s}M \hookrightarrow W^s P^r M$ is

$$J^{r+s}TM \hookrightarrow J^s TM \times_{J^s TM} J^s J^r TM\,,$$

where we consider both the jet projection $\pi^{r+s}_s\colon J^{r+s}TM \to J^s TM$ and the canonical injection $J^{r+s}TM \hookrightarrow J^s J^r TM$. According to [1], $(\tilde{t}_2)_{P^r M}\colon J^s P^r M \to F_2 P^r M$ induces a principal bundle morphism

$$W^s P^r M = P^s M \times_M J^s P^r M \to P^s M \times_M F_2 P^r M = W^{F_2} P^r M\,.$$

One verifies easily that its algebroid form

$$(14)\qquad J^s TM \times_{J^s TM} J^s J^r TM \to J^s TM \times_{F_2 TM} F_2 J^r TM$$

is determined by $(\tilde{t}_2)_{J^r TM}\colon J^s J^r TM \to F_2 J^r TM$. So we have

$$(15)\qquad \begin{aligned} L(W^{F_2\circ F_1}P) &= J^{r+s}TM \times_{F_2F_1TM} F_2F_1LP \\ &\hookrightarrow J^s J^r TM \times_{F_2F_1TM} F_2F_1LP \\ &\to (J^s TM \times_{F_2TM} F_2 J^r TM) \times_{F_2F_1TM} F_2F_1LP \\ &= J^s TM \times_{F_2TM} F_2(LW^{F_1}P) = L\big(W^{F_2}(W^{F_1}P)\big)\,. \end{aligned}$$

Using (12)–(15), we deduce

Proposition 1. *The algebroid homomorphism*

$$(16)\qquad L(W^{F_2\circ F_1}P) \to L\big(W^{F_2}(W^{F_1}P)\big)$$

corresponding to (11) *is the composition of all arrows in* (15)*.*

2. Reductions of W^1P. First we summarize our results from [8] on the torsion of connections on $W^r P$ in the case $r = 1$. We consider the principal bundle $W^1P = P^1M \times_M J^1P$ with structure group $W^1_m G = G^1_m \rtimes T^1_m G$ and the canonical one-form $\theta_1\colon TW^1P \to \mathbb{R}^m \times \mathfrak{g}$. For a connection Γ on W^1P, the torsion is defined to be the covariant exterior differential $D_\Gamma \theta_1$. This can be interpreted as a map

$$(17)\qquad \{D_\Gamma\theta_1\}\colon W^1P \to (\mathbb{R}^m \times \mathfrak{g}) \otimes \Lambda^2\mathbb{R}^{m*}\,.$$

On the other hand, we have $L(W^1P) = J^1(LP)$. Since the bracket $[\![\ ,\ ]\!]$ of LP is a first order differential operator, it induces the so-called truncated bracket

$$[\![\ ,\ ]\!]_1 \colon J^1LP \times_M J^1LP \to LP\,,$$

[8]. If we pass to the algebroid form $\gamma\colon TM \to J^1LP$ of Γ, we introduce $\tau\gamma\colon TM \times_M TM \to LP$ by

$$\tau\gamma(Z_1, Z_2) = [\![\gamma Z_1, \gamma Z_2]\!]_1\,, \qquad (Z_1, Z_2) \in TM \times_M TM\,. \tag{18}$$

This is a section of $LP \otimes \Lambda^2 T^*M$, what is a fiber bundle associated to W^1P with standard fiber $(\mathbb{R}^m \times \mathfrak{g}) \otimes \Lambda^2\mathbb{R}^{m*}$. So the frame form of $\tau\gamma$ is a map

$$\{\tau\gamma\}\colon W^1P \to (\mathbb{R}^m \times \mathfrak{g}) \otimes \Lambda^2\mathbb{R}^{m*}\,. \tag{19}$$

In [8], we deduced

$$\{D_\Gamma \theta_1\} = \frac{1}{2}\{\tau\gamma\}\,. \tag{20}$$

Remark. The coordinate formula for $\tau\gamma$ can be found in [8]. We find remarkable that this formula implies directly the following assertion. If Γ_1 and Γ_2 are two torsion-free connections on W^1P over the same connection on P, then every connection of the pencil $t\Gamma_1 + (1-t)\Gamma_2$, $t \in \mathbb{R}$, is also torsion-free.

Consider a reduction $Q \subset W^1P$ to a subgroup $H \subset W^1_mG$. In [4], Q is said to be a generalized G-structure. Write $\theta_Q\colon TQ \to \mathbb{R}^m \times \mathfrak{g}$ or $[\![\ ,\ ]\!]_Q\colon LQ \times_M LQ \to LP$ for the restriction of θ_1 or $[\![\ ,\ ]\!]_1$, respectively, and $i_Q\colon Q \to W^1P$ for the injection. A connection Γ on Q is canonically extended into a connection $\bar\Gamma$ on W^1P. Clearly, we have

$$D_\Gamma \theta_Q = i_Q^*(D_{\bar\Gamma}\theta_1)\,. \tag{21}$$

Further, for the algebroid form $\gamma\colon TM \to LQ$ of Γ, we define

$$\tau\gamma(Z_1, Z_2) = [\![\gamma Z_1, \gamma Z_2]\!]_Q\,, \qquad (Z_1, Z_2) \in TM \times_M TM\,.$$

Analogously to (19), we have its frame form

$$\{\tau\gamma\}\colon Q \to (\mathbb{R}^m \times \mathfrak{g}) \otimes \Lambda^2\mathbb{R}^{m*}$$

satisfying

$$\{\tau\gamma\} = \{\tau\bar\gamma\} \circ i_Q\,. \tag{22}$$

Then (20)–(22) imply

Proposition 2. *Both approaches to the torsion of connections on Q are related by*

$$\{D_\Gamma \theta_Q\} = \frac{1}{2}\{\tau\gamma\} \colon Q \to (\mathbb{R}^m \times \mathfrak{g}) \otimes \Lambda^2 \mathbb{R}^{m*} \,. \tag{23}$$

Further we need the basic facts concerning the prolongation of generalized G-structures. We define the second nonholonomic prolongation $\tilde{W}^2 P = W^1(W^1 P)$. By [4], $\tilde{W}^2 P = \tilde{P}^2 M \times_M \tilde{J}^2 P$, where $\tilde{P}^2 M$ is the second nonholonomic frame bundle of M and $\tilde{J}^2 P = J^1(J^1 P)$. The second semiholonomic prolongation $\bar{W}^2 P \subset \tilde{W}^2 P$ can be defined as $\bar{P}^2 M \times_M \bar{J}^2 P$. We have $W^1 Q \subset \tilde{W}^2 P$ and $\beta \colon W^1 Q \to Q$ is always surjective. According to [4], Q is called semiprolongable or prolongable, if the restriction of β to $W^1 Q \cap \bar{W}^2 P$ or $W^1 Q \cap W^2 P$ is also surjective, respectively. Write $Q_0 = \beta Q$ and $H_0 = \beta H$. In [4], we deduced

Lemma. *Q is semiprolongable, if and only if $Q \subset W^1(Q_0)$ or, equivalently, the values of θ_Q lie in $\mathbb{R}^m \times \mathfrak{h}_0$.*

Thus, if Q is semiprolongable, then (23) holds with the additional property that the values lie in $(\mathbb{R}^m \times \mathfrak{h}_0) \otimes \Lambda^2 \mathbb{R}^{m*}$.

For every $X \in W^1 P$, $X = (u, U)$, we have $U \circ u \in T^1_m P$. Hence X is identified with an m-dimensional subspace $\lambda(X) \subset TP$. So every $Z \in \tilde{W}^2 P$ is identified with $\lambda(Z) \subset TW^1 P$. According to Proposition 5 of [4], $Z \in \bar{W}^2 P$ satisfies

$$Z \in W^2 P \quad \text{if and only if} \quad d\theta_1 \mid \lambda(Z) = 0 \,. \tag{24}$$

This implies an assertion analogous to the classical theory of G-structures. Write $\pi \colon Q \to M$, $\pi_1 \colon Q \to P^1 M$ and $\pi_2 \colon W^1 Q \to J^1 Q$ for the canonical projections.

Proposition 3. *Let Q be semiprolongable. If Q admits a torsion-free connection, then Q is prolongable. Conversely, if Q is prolongable, then for every $x \in M$ there exists a neighbourhood U and a torsion-free connection on $\pi^{-1}(U)$.*

Proof. Let $\Gamma \colon Q \to J^1 Q$ be a torsion-free connection. By (24), the rule

$$X \mapsto \big(\pi_1(X), \Gamma(X)\big) \,, \qquad X \in Q$$

is a section $Q \to W^1 Q \cap W^2 P$, so that Q is prolongable. Conversely, let $\Sigma \colon Q \to W^1 Q \cap W^2 P$ be a section. For every section $\varrho \colon U \to Q$, the map $\pi_2 \circ \Sigma \circ \varrho \colon U \to J^1 Q$ is canonically extended into a connection on $\pi^{-1}(U)$. This connection is torsion-free due to the fact that θ_1 is a pseudo-tensorial form, [9, p.155]. □

3. Torsions on certain gauge-like prolongations. If E is a fiber product preserving bundle functor on $\mathcal{FM}_m$ satisfying $E \subset J^1 \circ F$, then $W^E P$ is a reduction of $W^1(W^F P)$ to a subgroup $K \subset W^1_m(W^A_H G)$. Write $E_0 = \beta E \subset F$ and $K_0 = \beta K \subset W^A_H G$. In general, we have $\theta_{EP} \colon T(EP) \to \mathbb{R}^m \times \mathfrak{w}^A_H G$. If EP is semiprolongable, then the values of θ_{EP} lie in $\mathbb{R}^m \times \mathfrak{k}_0$. The simpliest case of such situation is $\beta E = F$. We are going to present some examples.

Example 1. We start with the functor $\tilde{J}^r$ of r-th nonholonomic jet prolongation of fibered manifolds, $\tilde{J}^r Y = J^1(\tilde{J}^{r-1} Y)$. More generally, our approach can be applied to an arbitrary functor S of r-th jet prolongation of fibered manifolds. In accordance with [7], this means a fiber product preserving bundle functor on $\mathcal{FM}_m$ satisfying $J^r \subset S \subset \tilde{J}^r$. In particular, J^r can be reduced to this situation by means of the canonical inclusion $J^r \subset J^1 \circ J^{r-1}$. A further well known example is the r-th semiholonomic prolongation $\bar{J}^r \subset J^1 \circ \bar{J}^{r-1}$. Clearly, the semiprolongability condition is satisfied in the last two cases.

Example 2. Another example of our type is the composition $J^r \circ F$ for arbitrary F. More generally, we can consider every composition $S \circ F$ with S from Example 1.

Some further functors can be studies in this way by using a suitable natural equivalence.

Example 3. In [1] it is deduced that for every fiber product preserving bundle functor E on $\mathcal{FM}_m$ and every vertical Weil functor V^B there exists a canonical natural equivalence

$$\varkappa \colon V^B \circ E \approx E \circ V^B \,. \tag{25}$$

If $E \subset J^1 \circ F$, then $\varkappa(V^B \circ E) \subset J^1 \circ (F \circ V^B)$. Hence we have the situation of Section 2. In addition, one deduces that $\beta E = F$ implies $\beta\big(\varkappa(V^B \circ E)\big) = F \circ V^B$ by using the standard Weil algebra manipulations from [1].

References

[1] M. Doupovec, I. Kolář, *Iteration of Fiber Product Preserving Bundle Functors*, Monatsh. Math. **134** (2001), 901–916.

[2] C. Ehresmann, *Les prolongements d'un espace fibré différentiable*, C.R. Acad.Sci. Paris **240** (1955), 1755–1757.

[3] L. Fatibene, M. Francaviglia, *Natural and Gauge Natural Formalism for Classical Fields Theories*, Kluwer, 2003.

[4] I. Kolář, *Generalized G-structures and G-structures of higher order*, Boll. Un. Mat. Ital., Suppl. fasc. **3** (1975), 249–256.

[5] I. Kolář, *A generalization of the torsion form*, Čas. pěst. math. **100** (1975), 284–290.

[6] I. Kolář, *Functorial prolongations of Lie algebroids*, In: Proceedings Conf. Prague 2004, Charles University, Prague (2005), 305–314.

[7] I. Kolář, *Weil Bundles as Generalized Jet Spaces*, In: Handbook of Global Analysis, Elsevier (2008), 625–665.

[8] I. Kolář, *Connections on principal prolongations of principal bundles*, In: Proceedings Conf. Olomouc 2007, World Scientific (2008), 279–291.

[9] I. Kolář, P. W. Michor, J. Slovák, *Natural Operations in Differential Geometry*, Springer 1993.

[10] I. Kolář, W. M. Mikulski, *On the fiber product preserving bundle functors*, Differential Geom. Appl. **11** (1999), 105–115.

[11] A. Kumpera, D. Spencer, *Lie equations I*, Princeton University Press, 1972.

Institute of Mathematics and Statistics
Masaryk University
Kotlářská 2, CZ 611 37 Brno
Czech Republic